Leptogenesis with Friedberg-Lee Symmetry

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We consider the $\mu - \tau$ symmetric Friedberg-Lee (FL) symmetry for the neutrino sector and show that a specific FL translation leads to the tribimaximal mixing pattern of the Maki-Nakagawa-Sakata (MNS) matrix. We also apply the symmetry to the type-I seesaw framework and address the baryon asymmetry of the universe through the leptogenesis mechanism. We try to establish a relation between the net baryon asymmetry and CP phases included in the MNS matrix.

Keywords: Leptogenesis; Flavor Symmetry; Neutrino Mixing.

1. Introduction

From the neutrino oscillation experiments, we currently know for sure that the neutrinos have tiny but non-zero masses and mix with each other through the Maki-Nakagawa-Sakata (MNS) mixing matrix. In particular, the mixing pattern of the MNS matrix is almost coincident with the so-called tribimaximal (TBM) mixing[1], given by

$$V_{TB} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & -\sqrt{3} \\ -1 & \sqrt{2} & \sqrt{3} \end{pmatrix} . \tag{1}$$

However, there is no definitive theory to generate the neutrino masses and TBM mixing yet. The (type-I) seesaw mechanism is one of the most plausible extensions of the standard model (SM) to produce tiny neutrino masses while as a bonus it can explain the baryon asymmetry of the universe (BAU) through the leptogenesis mechanism[2]. In the leptogenesis scenario, CP asymmetry is an essential ingredient and it is related to the Dirac and Majorana phases in the MNS matrix. Nevertheless, it is usually very hard to connect them directly because the model also includes some high-energy phases[3,4], which are associated with the heavy right-handed Majorana neutrinos, and the CP asymmetry of the leptogenesis depends on the phases as well. In order to establish a direct relation between the CP asymmetry and phases in the MNS matrix, we need to reduce as many complex parameters as possible in the model. In Refs. 5, 6, a family symmetry is used to minimize the number of arbitrary parameters in the Yukawa sector. Another possibility along this direction

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is to consider the so-called two-right-handed neutrino (2RHN) seesaw model[7,8], in which the number of parameters is less numerous than the ordinary seesaw model. In addition, spontaneous[9] and dynamical[10] CP violating approaches have been proposed.

In this talk, we introduce the Friedberg-Lee (FL) symmetry[11,12,13] for the neutrino sector and show our results based on Ref. 14. The FL symmetry is a translational (hidden) family symmetry and some detailed analyses for the neutrino sector have been discussed in Refs. 15, 16, 17, 18 and 19. Remarkably, as pointed out in Ref. 20, the introduction of the FL symmetry to the right-handed neutrinos (RHNs) suggests the existence of a non-interacting massless RHN, and the theory comes down to the 2RHN seesaw model. This, in fact, motivates us to examine the leptogenesis in the context of the FL symmetry. We will consider the $\mu - \tau$ symmetric FL symmetry[21] and show that the TBM mixing can be explained by the specific pattern of the FL translation. In particular, we apply our scheme to the seesaw mechanism and address the BAU via the leptogenesis mechanism.

2. Friedberg-Lee Symmetry and Neutrino Mixing

We start our discussion with the Lagrangian of the Majorana neutrino mass term

$$-\mathcal{L}_\nu = \overline{\nu^c}_i \mathcal{M}^\nu_{ij} \nu_j + h.c. \,, \tag{2}$$

where the subscripts i and j stand for family indices. Here, we take $\mathcal{M}^\nu$ as a real matrix and consider the diagonal basis of the charged leptons. In this basis, we impose the FL symmetry on the Majorana neutrinos as follows

$$\nu_i \to \nu_i^{'} = \nu_i + (1, \eta, \eta\xi)^T z \,, \tag{3}$$

where z is a space-time independent Grassmann parameter, $z^2 = 0$, and η and ξ are c-numbers. The mass matrix takes the form

$$\mathcal{M}^\nu = \begin{pmatrix} B\eta^2 + C & -B\eta & -C/(\eta\xi) \\ -B\eta & A\xi^2 + B & -A\xi \\ -C/(\eta\xi) & -A\xi & A + C/(\eta\xi)^2 \end{pmatrix} . \tag{4}$$

We note that the mass matrix has one zero-eigenvalue[19], which is ensured by the FL symmetry[a]. According to the procedure in Ref. 12, we have assumed the relation $C = B\eta^2\xi^2$. Consequently, the MNS matrix can be expressed with only η and ξ:

$$V_{MNS} = \begin{pmatrix} \cos\sigma & -\sin\sigma & 0 \\ \sin\sigma\cos\rho & \cos\sigma\cos\rho & -\sin\rho \\ \sin\sigma\sin\rho & \cos\sigma\sin\rho & \cos\rho \end{pmatrix} \tag{5}$$

[a] In general, the FL symmetry leads to one zero-eigenvalue because the mass matrix needs to satisfy the condition $(1, \eta, \eta\xi)_i M_{ij} = 0$ to keep the invariance. In other words, the three-dimensional vector $(1, \eta, \eta\xi)$ corresponds to the eigenvector of the zero-eigenvalue.

with $\eta = \tan\sigma\cos\rho$ and $\xi = \tan\rho$. That is, the pattern of the neutrino mixing is governed by the FL symmetry in Eq. (3). Thus, by comparing Eq. (5) with the neutrino oscillation data, we can deduce the values of η and ξ. In particular, the TBM mixing pattern corresponds to $\eta = -1/2$ and $\xi = 1$. Interestingly, in this case, the relation assumed above becomes $C = B/4$ and it can be realized by imposing the $\mu-\tau$ symmetry. Namely, the FL symmetry, with $\eta = -1/2$ and $\xi = 1$, combined with the $\mu - \tau$ symmetry can naturally lead to the TBM mixing. So, we define the symmetry as follows

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} z \tag{6}$$

and call it twisted FL symmetry[21]. Note that the shift part of Eq. (6) is multiplied by a factor of -2 because the overall factor is irrelevant in this discussion.

3. Twisted Friedberg-Lee Symmetric Seesaw Model

3.1. *Framework of model*

We apply the twisted FL symmetry to the conventional (type-I) seesaw framework with three RHNs. The relevant Lagrangian is given by

$$-\mathcal{L}_{\text{seesaw}} = Y_D \bar{L}_L \tilde{H} \nu_R + \frac{1}{2} M_R \overline{\nu^c}_R \nu_R + h.c.\ , \tag{7}$$

where we have omitted family indices. We assume the diagonal charged lepton mass matrix again and impose the twisted FL symmetry on both the right- and left-handed neutrinos. Due to the symmetry, the Majorana mass matrix takes the form

$$M_R = \begin{pmatrix} B/2 & B/2 & B/2 \\ B/2 & A+B & -A \\ B/2 & -A & A+B \end{pmatrix}. \tag{8}$$

In addition to the twisted FL symmetry, we introduce a Z_2 symmetry for the lepton doublet and charged singlet of the first family in order to reproduce a realistic neutrino mass hierarchy. As a result, the Dirac mass matrix is given by

$$Y_D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & -\alpha \\ 0 & -\alpha & \alpha \end{pmatrix}. \tag{9}$$

The Majorana mass matrix in Eq. (8) can be diagonalized by the TBM matrix in Eq. (1), so that

$$\begin{aligned} D_R \equiv (PV_{TB}^T) M_R (V_{TB} P) &= \text{diag}(M_1, M_2, M_3) \\ &= \text{diag}(0, 3/2|B|, |2A+B|), \end{aligned} \tag{10}$$

where $P = \mathrm{diag}(1, e^{i\phi_R/2}, 1)$ is a diagonal phase matrix of the RHNs. In this basis, the Dirac mass matrix in Eq. (9) becomes

$$Y_R \equiv Y_D V_{TB} P = \sqrt{2}\alpha \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}. \tag{11}$$

Note that α can always be real by suitable redefinitions of the left-handed leptons. As pointed out in Ref. 20, in this basis, the RHN of the first family can be regarded as a non-interacting massless neutrino. By omitting this field, we can move to 3×2 dimensional Dirac mass matrix basis and rewrite Eq. (11) as

$$Y_R = \sqrt{2}\alpha \begin{pmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix}, \tag{12}$$

with $D_R = \mathrm{diag}(M_2, M_3)$. The mass matrix of the light neutrinos is written as

$$\mathcal{M}^\nu = v^2 Y_R D_R^{-1} Y_R^T = \frac{2\alpha^2 v^2}{M_3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}. \tag{13}$$

This matrix can be diagonalized with only one maximal angle and has only one non-zero eigenvalue. Hence, there are two interacting and one non-interacting massless neutrinos and no CP violating phase in the MNS matrix. Clearly, it is inconsistent with the experimental data of existing at least two massive light neutrinos and large mixing angles.

In order to obtain a realistic model, we introduce symmetry breaking terms in Eq. (9), given by

$$Y_D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & -\alpha \\ 0 & -\alpha & \alpha \end{pmatrix} + \begin{pmatrix} \frac{1}{4}\Delta & \frac{1}{2}\Delta & 0 \\ \frac{1}{2}\Delta & \Delta & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{14}$$

Note that the breaking terms violate both the permutation symmetry in Eq. (6) and the Z_2 symmetry, but still preserve the translational symmetry so that the first family light neutrino remains massless. Note also that although we could introduce breaking terms for the Majorana mass matrix as well, we only focus on the effect from the Dirac mass matrix in the following discussions.

In the diagonal basis of the RHNs, the Dirac mass matrix can again be reduced to an 3×2 dimensional matrix and becomes

$$Y_R = \frac{1}{2} \begin{pmatrix} \frac{\sqrt{3}}{2}\Delta e^{i\phi_R/2} & -\frac{\sqrt{2}}{2}\Delta \\ \sqrt{3}\Delta e^{i\phi_R/2} & -2\sqrt{2}\alpha - \sqrt{2}\Delta \\ 0 & 2\sqrt{2}\alpha \end{pmatrix}. \tag{15}$$

In what follows, we consider the basis where α is real but Δ is complex, $\Delta \equiv |\Delta| e^{i\phi_\Delta}$. The mass matrix of the light neutrinos is given by

$$\mathcal{M}^\nu = \frac{v^2}{M_3}\left[2\alpha^2\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{pmatrix} + \frac{\alpha\Delta}{2}\begin{pmatrix}0 & 1 & -1\\ 1 & 4 & -2\\ -1 & -2 & 0\end{pmatrix} + \frac{\Delta^{'2}}{8}\begin{pmatrix}1 & 2 & 0\\ 2 & 4 & 0\\ 0 & 0 & 0\end{pmatrix}\right], \quad (16)$$

where the second and third terms are responsible for the deviations from the tribimaximal mixing with

$$\Delta^{'2} = |\Delta|^2 e^{2i\phi_\Delta}\left[1 + \frac{3}{2}\frac{M_3}{M_2}e^{i\phi_R}\right], \quad (17)$$

while ϕ_Δ and ϕ_R generate CP violation in the MNS matrix. Here, we define the MNS matrix as

$$V_{MNS} = V_{TB}\ \delta V\ \Omega = \frac{1}{\sqrt{6}}\begin{pmatrix}2 & \sqrt{2} & 0\\ -1 & \sqrt{2} & -\sqrt{3}\\ -1 & \sqrt{2} & \sqrt{3}\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & c_\theta & s_\theta e^{-i\delta}\\ 0 & -s_\theta e^{i\delta} & c_\theta\end{pmatrix}\Omega, \quad (18)$$

where $s_\theta = \sin\theta$ ($c_\theta = \cos\theta$) with

$$\tan 2\theta = -\frac{\sqrt{6}(\alpha\Delta + \Delta^{'2}/4)e^{i\delta}}{(4\alpha^2 + 2\alpha\Delta + \Delta'^2/4)e^{2i\delta} - 3/8\Delta'^2} \equiv -\frac{\mathcal{I}e^{i\delta}}{\mathcal{J}e^{2i\delta} - \mathcal{K}}\ , \quad (19)$$

δ is a Dirac-type CP phase which has to satisfy

$$\delta = -\frac{i}{2}\ln\left[\frac{\mathcal{I}\mathcal{J}^* + \mathcal{I}^*\mathcal{K}}{\mathcal{I}^*\mathcal{J} + \mathcal{I}\mathcal{K}^*}\right], \quad (20)$$

to guarantee the right hand side of Eq. (19) to be real, and $\Omega = \mathrm{diag}(1, e^{i\gamma/2}, 1)$ is a diagonal Majorana-type CP phase matrix. Note that relations between our definitions of the Dirac and Majorana phases and those in Particle Data Group[22] (PDG) are given in Ref. 14. The mixing angles are given by

$$\sin^2\theta_{13} = \frac{1}{3}s_\theta^2, \quad (21)$$

$$\sin^2\theta_{12} \simeq \frac{1}{3}(1 - s_\theta^2), \quad (22)$$

$$\sin^2\theta_{23} \simeq \frac{1}{2} - \frac{1}{6}s_\theta^2 - \frac{\sqrt{6}}{3}s_\theta c_\theta\cos\delta. \quad (23)$$

The mass matrix in Eq. (16) is diagonalized by Eq. (18), leading to the masses of

the light neutrinos to be

$$m_1 = 0, \tag{24}$$

$$m_2 = \frac{v^2}{M_3}\left|4\alpha^2 s_\theta^2 e^{2i\delta} + \alpha\Delta(\sqrt{6}s_\theta c_\theta e^{i\delta} + 2s_\theta^2 e^{2i\delta}) + \frac{\Delta'^2}{4}(\sqrt{6}s_\theta c_\theta e^{i\delta} + s_\theta^2 e^{2i\delta} + 3/2c_\theta^2)\right|, \tag{25}$$

$$m_3 = \frac{v^2}{M_3}\left|4\alpha^2 c_\theta^2 + \alpha\Delta(-\sqrt{6}s_\theta c_\theta e^{-i\delta} + 2c_\theta^2) + \frac{\Delta'^2}{4}(-\sqrt{6}s_\theta c_\theta e^{-i\delta} + 3/2s_\theta^2 e^{-2i\delta} + c_\theta^2)\right|. \tag{26}$$

The Majorana phase is given by

$$\gamma = -\gamma_2 + \gamma_3\,, \tag{27}$$

where

$$\sin\gamma_2 = \frac{\mathrm{Im}[m_2]}{|m_2|}, \quad \sin\gamma_3 = \frac{\mathrm{Im}[m_3]}{|m_3|}. \tag{28}$$

From Eqs. (20), (25), (26), (27) and (28), one can see that the Dirac and Majorana phases are originated from ϕ_R and ϕ_Δ.

3.2. *CP violation*

Our model possesses two CP violating phases: ϕ_R and ϕ_Δ, plus four real parameters: α, $|\Delta|$, $|2A+B|$ and $|B|$. These six theoretical parameters can be fixed by six physical quantities. In our calculations, we will use the following best-fit values with 1σ errors:

$$\Delta m_{21}^2 = (7.65^{+0.23}_{-0.20}) \times 10^{-5}\ \mathrm{eV}^2, \quad |\Delta m_{31}^2| = (2.40^{+0.12}_{-0.11}) \times 10^{-3}\ \mathrm{eV}^2, \tag{29}$$

$$\sin^2\theta_{12} = 0.304^{+0.022}_{-0.016}, \quad \sin^2\theta_{23} = 0.50^{+0.07}_{-0.06}, \tag{30}$$

from Ref. 23 and

$$\sin^2\theta_{13} = 0.02 \pm 0.01 \tag{31}$$

from the global analysis in Ref. 24, and take

$$M_3 = 8.0 \times 10^{10}\ \mathrm{GeV}, \quad M_3/M_2 = 5 \tag{32}$$

as input parameters. As we will discuss later, the masses of RHNs are determined to account for the measured value of the BAU.

By using Eqs. (21) - (23) and 1σ values of $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$, we can estimate the range of δ to be

$$67^\circ < \delta < 122^\circ\,. \tag{33}$$

In Fig. 1, we show the numerical result of δ as a function of $\sin^2\theta_{23}$. As can be seen

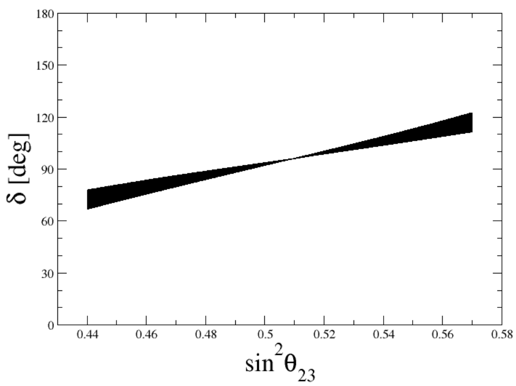

Fig. 1. The Dirac phase δ as a function of $\sin^2\theta_{23}$ with $M_3 = 8.0\times 10^{10}$ GeV and $M_3/M_2 = 5$.

from the figure, the result is coincident with Eq. (33) very well.

In contrast, γ has a wide allowed range and it could have an impact on the neutrinoless double β decay due to the effective Majorana mass

$$< m_{ee} >= \left|\sum_{i=1}^{3} m_i (V_{MNS})^2_{1i}\right| \tag{34}$$

since the Dirac phase as well as the individual neutrino mass can be determined within some ranges in our model. In Fig. 2, we give the effective mass as a function of γ. Unfortunately, the predicted values of $< m_{ee} >$ in our model are around $(2.2-4.1)\times 10^{-3}$ eV, which are too small to be detected in the current and upcoming experiments. For instance, the order of the present sensitivity at the CUORICINO experiment is 10^{-1} eV, while that of the proposed CUORE detector is 10^{-2} eV.[25] Nevertheless, we would like to emphasize that more dedicated experiments in future are needed in order to determine the Majorana phase.

Finally, we would like to briefly remark on the possibility to test our model. As our model predicts the following novel relation

$$\sin^2\theta_{13} \simeq 1/3 - \sin^2\theta_{12} \tag{35}$$

based on Eqs. (21) and (22), more precise determinations of mixing angles would provide us a chance to rule out or confirm the model in future. For instance, the smaller value of $\sin^2\theta_{12}$, which is $\sin^2\theta_{12} = 0.304^{+0.000}_{-0.016}$, results in $\sin^2\theta_{13} = 0.0293 \sim 0.0453$ which goes beyond the 1σ ranges given in Ref. 23 and the global analysis in Ref. 24. On the other hand, the larger value of $\sin^2\theta_{12} = 0.304^{+0.022}_{-0.000}$ corresponding to $\sin^2\theta_{13} = 0.0073 \sim 0.0293$ is well coincident with Refs. 23 and 24.

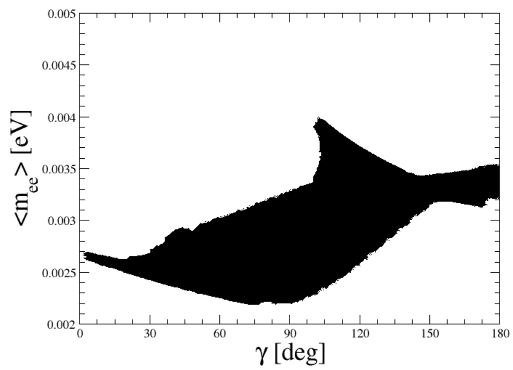

Fig. 2. The effective mass $< m_{ee} >$ as a function γ with $M_3 = 8.0 \times 10^{10}$ GeV and $M_3/M_2 = 5$.

4. Leptogenesis

As discussed in the previous section, our model results in non-zero values of δ and $\sin\theta_{13}$ as shown in Eq. (33) and Eq. (35) with Eq. (30), respectively. This means that the CP symmetry is always violated in the lepton sector even if there is no Majorana phase γ. In this section, we consider the unflavored leptogenesis mechanism[b] via the out-of-equilibrium decays of the heavy RHNs. The CP violating parameter in the leptogenesis due to the i-th heavy RHN decays is written as

$$\varepsilon_i = -\frac{1}{8\pi} \sum_{j \neq i} \frac{\mathrm{Im}[(Y_R^\dagger Y_R)^2_{ji}]}{(Y_R^\dagger Y_R)_{ii}} \, F\left(\frac{M_j^2}{M_i^2}\right), \tag{36}$$

where $i, j = 2$ or 3, $F(x)$ is given by

$$F(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln\frac{1+x}{x}\right], \tag{37}$$

Y_R is the Dirac mass matrix in the diagonal basis of the right-handed neutrinos and charged leptons, given in Eq. (15), with the first (second) column referred as $Y_{R_{j2}}$ ($Y_{R_{j3}}$). The dilution factor κ_i is approximately given by[29]

$$\kappa_i \simeq \frac{0.3}{r_i(\ln r_i)^{0.6}}\ , \tag{38}$$

where

$$r_i = \frac{\Gamma_i}{H|_{T=M_i}} = \frac{M_{pl}}{1.66\sqrt{g_*}M_i^2}\frac{(Y_R^\dagger Y_R)_{ii}}{16\pi}M_i \tag{39}$$

[b] The importance of the flavor effects is discussed in Refs. 26, 27 and 28.

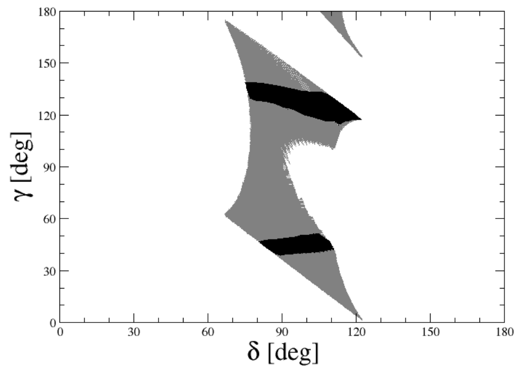

Fig. 3. Allowed regions in $\delta - \gamma$ plane with $M_3 = 8.0 \times 10^{10}$ GeV and $M_3/M_2 = 5$, where the gray and black regions correspond to those fitted by only the neutrino oscillation and with WMAP data at 1σ, respectively.

with $M_{pl} = 1.22 \times 10^{19}$ GeV and $g_* = 106.75$. The net BAU is found to be

$$\eta_B = \frac{n_B}{n_\gamma} = 7.04 \frac{\omega}{\omega - 1} \frac{\kappa_2 \varepsilon_2 + \kappa_3 \varepsilon_3}{g_*}, \tag{40}$$

where $\omega = 28/79$. Here, instead of showing some complex analytic calculations, we only give the numerical results. In Fig. 3, we show the allowed regions in $\delta - \gamma$ plane with $M_3 = 8.0 \times 10^{10}$ GeV and $M_3/M_2 = 5$, where the gray and black regions represent to those fitted by only the neutrino oscillation and with 1σ WMAP[30] bound $\eta_B = (6.1^{+0.2}_{-0.2}) \times 10^{10}$, respectively. One can easily see that there is an explicit connection between the leptogenesis and the phases in the MNS matrix. Especially, the Majorana phase is closely related to the leptogenesis and limited to two narrow regions.

5. Conclusion

We have considered the twisted FL symmetry which can successfully generate the TBM neutrino mixing. We have applied the symmetry to the seesaw framework and shown a specific model which is well consistent with current neutrino oscillation data. We have studied the BAU through the leptogenesis mechanism in the model and found that the net baryon asymmetry is directly connected with the CP violating Dirac and Majorana phases in the MNS matrix.

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